

8

Inference for Proportions

Calculator Note 8A: Simulating Binomial Experiments

You can use your calculator to simulate the reasonably likely outcomes for a binomial experiment, as you saw in Calculator Note 7B. For example, to simulate the number of successes in a sample of size 40 from a population with 60% successes, use the random binomial generator command, `randBin(`. You find the command by pressing `MATH`, arrowing over to `PRB`, and selecting `7:randBin(`. The command's syntax is `randBin(number of trials, probability of success, number of simulations)`; if `number of simulations` is omitted, the default is 1. For example, `randBin(40,.6)` returns a single number that represents the number of successes in a sample of 40. The command `randBin(40,.6,100)` returns the number of successes for 100 simulations, which can be stored in a list.

```
MATH NUM CPX PRB
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

```
randBin(40,.6)
26
```

```
randBin(40,.6,100)
0)→L1
(24 27 22 21 21...
```

Calculator Note 8B: Calculating the Critical Value z^* for a Given Confidence Level or Significance Level `invNorm(`

To calculate the value of z^* that corresponds to a particular level of confidence, you must first determine the proportion of values at or below that value. For example, if you want z^* for a 95% confidence interval, you must realize that the middle 95% of the data lie between points that cut off an area of 0.025 and an area of 0.975 in the lower tail of the standard normal distribution. Because the distribution is symmetric, you need only the values of z that correspond to a lower tail area of 0.975. You use the command `invNorm`. You find this command by pressing `2ND` `[DISTR]` and selecting `3:invNorm(` from the `DISTR` menu. The command's syntax is `invNorm(area, μ , σ)`; if μ and σ are omitted, the default values are 0 and 1, respectively. The command `invNorm(.975)` calculates 1.959963986, so the critical values z^* are approximately ± 1.96 .

```
invNorm(.975)
1.959963986
```

The same procedure is used for calculating the critical value for a significance test. Here it is important to pay attention to whether the test is one-sided or two-sided. For example, a two-sided significance test of a proportion would use the same critical values for a 5% significance level as for a 95% confidence

interval. However, a one-sided significance test at the 5% level would use the value of z^* that corresponds to an area of either 5% or 95%, depending on the direction of the test. For example, in the case that the alternative hypothesis is $p > p_0$, you would use $\text{invNorm}(.95)$, which is 1.645.

```
invNorm(.05)
-1.644853626
invNorm(.95)
1.644853626
```

Calculator Note 8C: Calculating Confidence Intervals 1-PropZInt

The TI-83 Plus and TI-84 Plus calculate a confidence interval for a proportion with the command 1-PropZInt. Find this command by pressing STAT , arrowing over to TESTS, and selecting A:1-PropZInt. Enter the number of successes in the sample, x , the sample size, n , and the level of confidence, C-Level. Arrow down and select Calculate to get the confidence interval and sample proportion. These screens show how to calculate the 90% confidence interval for a population from which a sample of 40 resulted in 24 successes. You find that the 90% confidence interval is approximately 47% to 73%.

```
EDIT CALC TESTS
0: 2-SampInt...
1: 1-PropZInt...
2: 2-PropZInt...
3: X2-Test...
4: 2-SampFTest...
5: LinRegTTest...
6: ANOVA
```

```
1-PropZInt
x:24
n:40
C-Level:.90
Calculate
```

```
1-PropZInt
(.47259,.72741)
p=.6
n=40
```

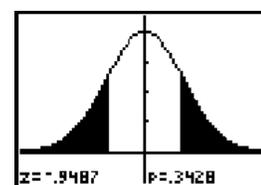
Calculator Note 8D: Calculating P-Values 1-PropZTest

The TI-83 Plus and TI-84 Plus conduct a significance test for a proportion and calculate the P -value with the command 1-PropZTest. You find the command by pressing STAT , arrowing over to TESTS, and selecting 5:1-PropZTest. At the prompts, enter the proportion under the null hypothesis, p_0 , the number of successes in the sample, x , and the sample size, n . (*Note: If you are given \hat{p} instead of x , calculate x by multiplying $\hat{p} \cdot n$ and rounding to the nearest integer.*) Select the type of test: two-tailed ($\neq p_0$) or one-tailed ($< p_0$ or $> p_0$). Arrow down and select Calculate to get the value of the test statistic, z , the P -value, p , the sample proportion, \hat{p} , and the sample size. Select Draw to get a shaded distribution. The screens here show the process for Jenny and Maya as shown on pages 495–496 of the student book.

```
EDIT CALC TESTS
1: 2-Test...
2: T-Test...
3: 2-SampZTest...
4: 2-SampFTest...
5: 1-PropZTest...
6: 2-PropZTest...
7: ZInterval...
```

```
1-PropZTest
P0:.5
x:17
n:40
PROPT: <P0 >P0
Calculate Draw
```

```
1-PropZTest
PROPT:.5
z=-.9486832981
p=.3427816815
p=.425
n=40
```



As outlined on pages 498–499 of the student book, statisticians have chosen clear and definitive guidelines for the presentation of a test of significance. It is critical that you understand the appropriate use of calculator technology within this presentation. You should use your calculator only after writing the name of the test, checking the criteria for its use, stating the hypotheses, and stating the test statistic. At that point, it is appropriate to have the calculator perform the test and calculate the P -value. Complete the significance test by stating whether or not you reject H_0 and writing a conclusion in context.

Calculator Note 8E: Confidence Interval for the Difference of Two Proportions 2-PropZInt

The TI-83 Plus and TI-84 Plus calculate the confidence interval for the difference of two proportions with the command 2-PropZInt. You find this command by pressing [STAT], arrowing over to TESTS, and selecting B:2-PropZInt. At the prompts, enter the number of successes in each sample, x_1 and x_2 , both sample sizes, n_1 and n_2 , and the level of confidence, C-Level. Arrow down and select Calculate to get the confidence interval, both sample proportions, and both sample sizes. Note that 2-PropZInt does not ask for the sample proportions. So, if you are given the proportions instead of the number of successes, multiply the proportions by the sample size and round to the nearest whole number. For instance, with the dog-ownership example on pages 520–521 of the student book, use $x_1 = 18711$ and $x_2 = 3800$.

```

EDIT CALC TESTS
6:2-PropZTest...
7:ZInterval...
8:TInterval...
9:2-SampZInt...
0:2-SampTInt...
A:1-PropZInt...
B:2-PropZInt...
    
```

```

2-PropZInt
x1:18711
n1:29700
x2:3800
n2:6786
C-Level:.95
Calculate
    
```

```

2-PropZInt
(.0572,.08305)
p1=.63
p2=.559976422
n1=29700
n2=6786
    
```

Calculator Note 8F: Significance Test for the Difference of Two Proportions 2-PropZTest

The TI-83 Plus and TI-84 Plus conduct a significance test for the difference of two proportions with the command 2-PropZTest. You find this command by pressing [STAT], arrowing over to TESTS, and selecting 6:2-PropZTest. The command behaves similarly to the 1-PropZTest explained in Calculator Note 8D. These screens show the calculations for the dress-code example on pages 533–534 of the student book. Note that the input requires the numbers of successes instead of proportions and that the output includes the test statistic, the P -value, the sample proportions, and the pooled estimate, \hat{p} .

```

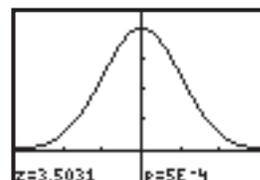
EDIT CALC TESTS
1:Z-Test...
2:T-Test...
3:2-SampZTest...
4:2-SampTInt...
5:1-PropZTest...
6:2-PropZTest...
7:ZInterval...
    
```

```

2-PropZTest
x1:156
n1:274
x2:115
n2:274
P1:5/24 <P2 >P2
Calculate Draw
    
```

```

2-PropZTest
P1#P2
z=3.503076375
P=4.6000571E-4
p1=.5693430657
p2=.4197080292
p=.4945255474
    
```



Calculator Note 8G: Types of Errors

The program ERRORS can help you understand Type II errors and power in a significance test for a proportion. The program graphically shows the effects of changing parameters on the probability of a Type II error and the power of the test.

- a. Run the program: press **PRGM** and select ERRORS from the EXEC menu. Press **ENTER**. Read the introductory screens, which emphasize that the program uses a normal approximation and a one-tailed test where $p > p_0$. Press **ENTER** after each screen.

```

ERRORS TESTING
A PROPORTION

PLEASE MAKE
P > P0

PRESS ENTER
    
```

```

ASSUME NORMAL
APPROXIMATION

ASSUME
ONE-TAILED TEST

PRESS ENTER
    
```

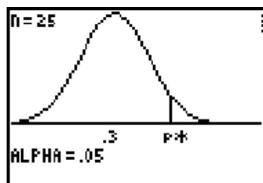
- b. At the prompts, enter the hypothesized proportion you are testing, p_0 , the true value of the population proportion, p , the sample size, and the significance level, α (ALPHA). The hypothesized proportion, p_0 , can be any value less than the true proportion. Press **ENTER** after each value.

For example, suppose you plan to take a sample of size 25 to test that $p = 0.3$. Although you don't know it, the true proportion is $p = 0.4$. The null hypothesis that $p = 0.3$ is false.

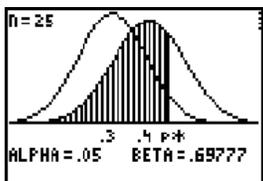
```

P0? .3
P? .4
SAMP SIZE? 25
ALPHA? .05
    
```

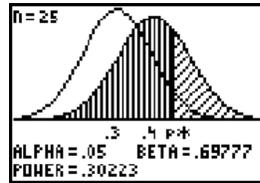
- c. The program first graphs a normal approximation of the sampling distribution for the hypothesized proportion p_0 . The critical value p^* is labeled. The null hypothesis will (correctly) be rejected if \hat{p} falls above this critical value. The null hypothesis will (incorrectly) not be rejected if \hat{p} falls below p^* .



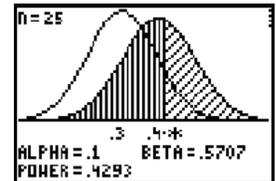
- d. Pressing **ENTER** graphs a normal approximation of the sampling distribution for the true population proportion. The tail is highlighted below the critical value p^* , which represents the probability of *not* rejecting this false null hypothesis. The area of this tail, called BETA, or β , represents the probability of a Type II error.



- e. Pressing **[ENTER]** again shades the second curve above the critical value. This area, $1 - \beta$, represents the power of the test. It represents the probability of rejecting this false null hypothesis.



- f. Pressing **[ENTER]** gives you the option to change the true population proportion, the sample size, or the significance level. Changing the parameters shows the effects on the probability of a Type II error. For example, if you increase the significance level, the probability of a Type II error decreases. (But if you were correct in the first place that $p = 0.3$, then you have increased the probability of a Type I error.)



In practice you don't know the value of p , so it is impossible to compute the probability of a Type II error and the power of the test. However, given a plausible value of p , the ERRORS program allows you to explore how the probability of a Type II error and the power of the test are affected by changes in α and the sample size.

```

PROGRAM:ERRORS
ClrHome
PlotsOff
FnOff
AxesOn
Disp "ERRORS TESTING"
Disp "A PROPORTION"
Disp ""
Disp "PLEASE MAKE"
Disp "P > P0"
Disp ""
Disp "PRESS ENTER"
Pause
ClrHome
Disp "ASSUME NORMAL"
Disp "APPROXIMATION"
Disp ""
Disp "ASSUME"
Disp "ONE-TAILED TEST"
Disp "PRESS ENTER"
Pause
ClrHome
Input "P0? ",P
Input "P? ",Q
If P>Q
Then
Disp "PLEASE MAKE"
Disp "P > P0"
Disp "P0 ",P
Input "P? ",Q
End
Input "SAMP SIZE? ",N
Input "ALPHA? ",A
Lbl D
invNorm(1-A,P,√(P(1-P)/N))→Z
"normalcdf(X,P,√(P(1-P)/N))"→Y1
"normalcdf(X,Q,√(Q(1-Q)/N))"→Y2
FnOff
P-3√(P(1-P)/N)→Xmin
Q+3√(Q(1-Q)/N)→Xmax
0→Xsc1
max(Y1(P),Y2(Q))+.02→Ymax
-Ymax/2→Ymin
    
```

(continued)

(PROGRAM: ERRORS continued)

```

0→Ysc1
ClrDraw
DrawF Y1
Text(0,0,"N= ",N)
Text(43,round(91(P-Xmin)/(Xmax-
Xmin),0)-2,P)
Text(43,round((91(Z-Xmin)/(Xmax-
Xmin)),0),"P*")
Text(50,0,"ALPHA = ",A)
Line(Z,0,Z,Y1(Z))
Pause
Text(43,round(91(Q-Xmin)/(Xmax-
Xmin),0)-2,Q)
Shade(0,Y2,Xmin,Z,1,2)
Line(Z,0,Z,Y2(Z))
normalcdf(-18699,Z,Q,√(Q(1-Q)/N))→B
round(B,5)→B
Text(50,47,"BETA = ",B)
Pause
Shade(0,Y2,Z,Xmax,4,4)
Text(57,0,"POWER = ",1-B)

```

```

Pause
ClrHome
Menu("CHANGE:", "P",A,"SAMP SIZE",B,
"ALPHA",C,"QUIT",E)
Lbl A
Input "NEW P? ",Q
If P ≥ Q
Then
Disp "PLEASE MAKE"
Disp "P > P0"
Disp "P0 ",P
Input "NEW P? ",Q
End
Goto D
Lbl B
Input "NEW SAMP SIZE? ",N
Goto D
Lbl C
Input "NEW ALPHA? ",A
Goto D
Lbl E
ClrHome

```