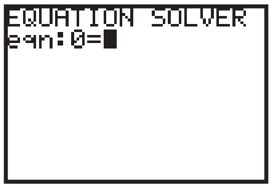
9 Inference for Means

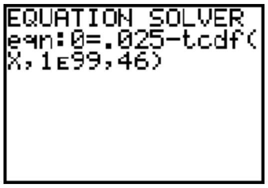
Calculator Note 9A: Using a Calculator to Find *t**

When a *t*-table does not provide a sufficiently exact confidence level or the needed degrees of freedom, the calculator provides several methods for finding the critical value *t**. One method uses the Equation Solver, which you find by pressing MATH and selecting **0:Solver** from the **MATH** menu. Follow these steps to find the value of *t** for a 95% confidence interval based on a sample of size 47. (Note that *df* = 46 is not in the *t*-table on page 826 of the student book.)

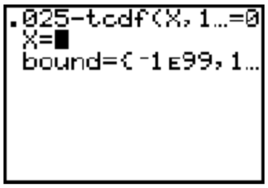
a. Start the Equation Solver. If the screen does not say **EQUATION SOLVER** and give you a place to edit the equation (**eqn:**), press the up arrow key once.

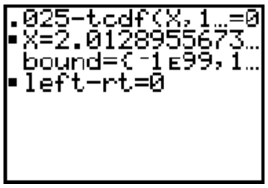


b. A 95% confidence interval means that you want a tail area of 0.025 on either side. Consequently, you need to find the value of *t** for which the cumulative distribution function equals 0.025 to the right of *t**. Find the cumulative distribution function by pressing 2nd [DISTR] and selecting **5:tcdf(** from the **DISTR** menu. The syntax is **tcdf(***lower bound, upper bound, degrees of freedom*). Enter the equation **0=.025−tcdf(X,1E99,46)**. Solving this equation for **X** gives the value of *t** with area 0.025 above it.

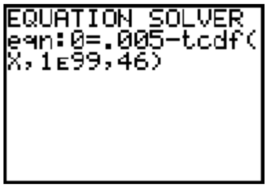


c. Press ENTER, place the cursor after **X=**, and press ALPHA [**SOLVE**]. The result indicates that *t** is approximately 2.013.





d. To modify the level of confidence or degrees of freedom, arrow up to the equation and you'll return to the Equation Editor screen.

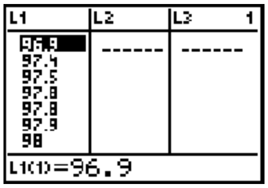


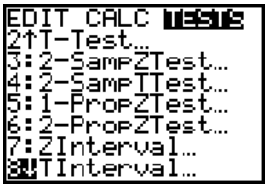
Calculator Note 9B: Calculating the Confidence Interval for a Mean **TInterval and ZInterval**

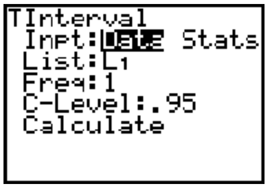
The TI-83 Plus and TI-84 Plus calculate the confidence interval for a mean when σ is unknown with the command **TInterval** and when σ is known with the command **ZInterval**. You find these commands by pressing STAT, arrowing over to **TESTS**, and selecting **8:Tinterval** or **7:ZInterval**. You can use sample data saved in a list or a frequency table saved in two lists, and the calculator will calculate the sample mean and standard deviation, or you can enter the statistics yourself.

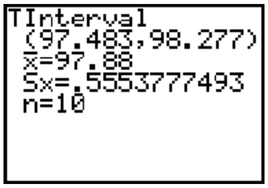
Calculating the Confidence Interval from a List of Data When σ Is Unknown

Enter your data into a list, say, L_1, or into a frequency table using, say, lists L_1 and L_2. Press STAT, arrow over to **TESTS**, and select **8:TInterval**. Select **Data**, enter the list number (and frequency list number if applicable), and specify the confidence level. Arrow down and select **Calculate** to get the confidence interval and sample statistics. These screens show the 95% confidence interval for the mean body temperature of men from pages 566–567 of the student book. The data for males from Display 9.3 were first saved in list L_1.



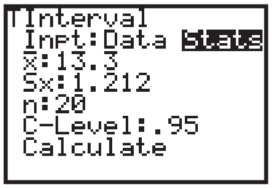


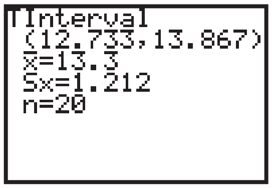




Calculating the Confidence Interval from Summary Statistics When σ Is Unknown

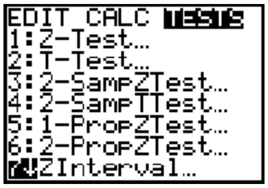
Press STAT, arrow over to **TESTS**, and select **8:TInterval**. Select **Stats**, enter the mean, standard deviation, and sample size, and specify the confidence level. Arrow down and select **Calculate** to get the confidence interval. These screens show the 95% confidence interval for a sample of size 20 with mean 13.3 and standard deviation 1.212.

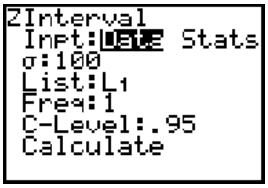


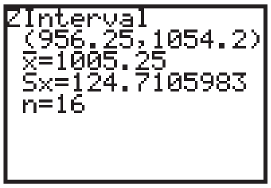


Calculating the Confidence Interval from a List of Data When σ Is Known

Enter your data into a list, say, L1, or into a frequency table using, say, lists L1 and L2. Press STAT, arrow over to **TESTS**, and select **7:ZInterval**. Select **Data**, enter the population standard deviation, σ, the list number (and frequency list number if applicable), and the confidence level, **C-Level**. Arrow down and select **Calculate** to get the confidence interval.

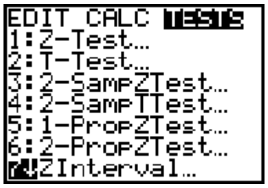


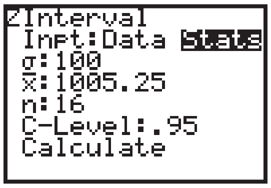


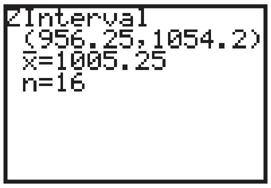


Calculating the Confidence Interval from Summary Statistics When σ Is Known

Press STAT, arrow over to **TESTS**, and select **7:ZInterval**. Select **Stats**, and enter the population standard deviation, σ, the sample mean, the sample size, and the confidence level, **C-Level**. Arrow down and select **Calculate** to get the confidence interval.

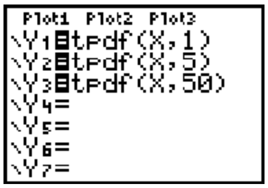


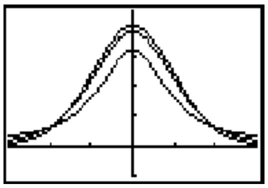




Calculator Note 9C: Graphing *t*-Distributions **tpdf(**

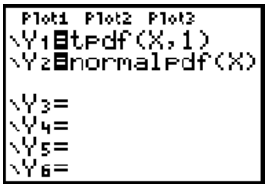
Using a TI-83 Plus or TI-84 Plus, you can visually explore the effect that the degrees of freedom has on the *t*-distribution. Use the *t* probability density function, **tpdf(**, which is found by pressing 2nd [**DISTR**] and selecting **4:tpdf(** from the **DISTR** menu. On the Y= screen, define functions in the form **tpdf(X,** *degrees of freedom*).

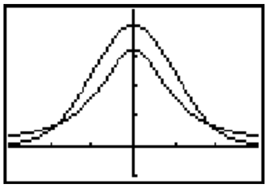




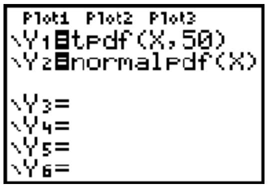
[−3, 3, 1, −0.1, 0.45, 0.1]

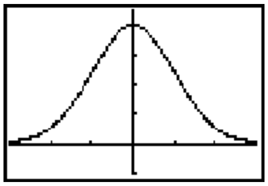
You can similarly compare a *t*-distribution to the standard normal distribution. This allows you to see that the *t*-distribution is "flatter" than the standard normal distribution, but as the degrees of freedom increases, the *t*-distribution approaches the standard normal distribution.





[−3, 3, 1, −0.1, 0.45, 0.1]

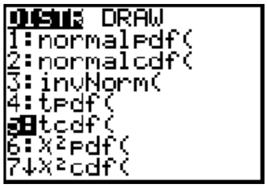


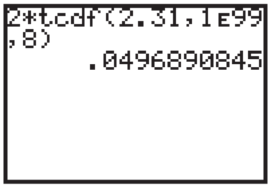


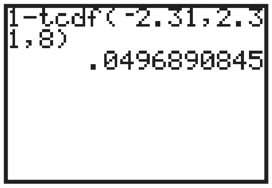
[−3, 3, 1, −0.1, 0.45, 0.1]

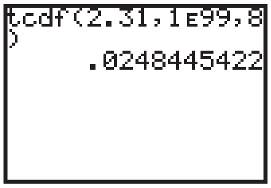
Calculator Note 9D: *P*-Values **tcdf(**

The TI-83 Plus and TI-84 Plus calculate the *P*-value given *t* and *df* with the command **tcdf(**. You find this command by pressing [2nd] **[DISTR]** and selecting **4:tcdf(** from the **DISTR** menu. Enter this command on the Home screen using the syntax **tcdf(***lower bound, upper bound, degrees of freedom*). For the mean weight of pennies example on page 585 of the student book, you can find the area in the two tails by entering either **2*tcdf(2.31,1E99,8)** or **1−tcdf(−2.31,2.31,8)**. To get the area in only one tail, you would enter **tcdf(2.31,1E99,8)**.





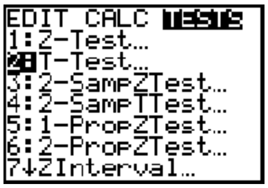


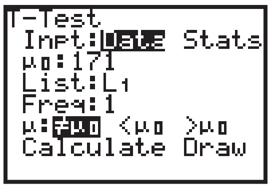


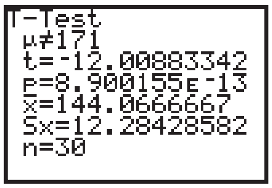
Calculator Note 9E: Significance Tests for a Mean **T-Test** and **Z-Test**

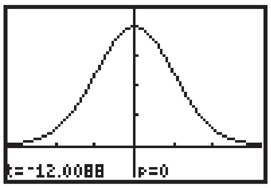
Significance Tests for a Mean When σ Is Unknown

The TI-83 Plus and TI-84 Plus conduct a significance test for a mean when σ is unknown with the command **T-Test**. You find the command by pressing [STAT], arrowing over to **TESTS**, and selecting **2:T-Test**. You can use data saved in lists or enter the sample mean, sample standard deviation, and sample size yourself. Either way, you must select whether the test is one-sided or two-sided. **Calculate** outputs the *t*-statistic, **t**, the *P*-value, **p**, and the sample statistics. **Draw** gives a shaded distribution. Here's the test for the mass of large bags of fries from pages 590−591 of the student book. The values vary slightly from those given in the text because these are calculated directly from the data, rather than using rounded values of the mean and standard deviation. Note that no shaded area is visible in the shaded distribution because the value of *t* is so far out in the tails.









Remember that whenever you make a written report about a *t*-test, you should report the number of degrees of freedom for the sampling distribution. The degrees of freedom specifies which *t*-distribution is the appropriate one for the test and should not be omitted, even though it is not part of the calculator's output.

Significance Tests for a Mean When σ Is Known

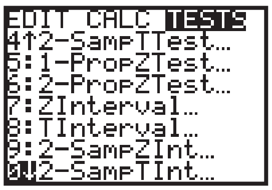
The TI-83 Plus and TI-84 Plus conduct a significance test for a mean with the command **Z-Test**. You find this command by pressing STAT, arrowing over to **TESTS**, and selecting **1:Z-Test**. As for **T-Test**, you can use sample data saved in a list or lists and the calculator will calculate the sample mean, or you can enter the sample mean and sample size as statistics. Either way, you must enter the population mean and population standard deviation and select whether the test is one-tailed or two-tailed. Select **Calculate** to get the test statistic, **z**, and the *P*-value, **p**, or select **Draw** to see the results with a shaded distribution. For a fixed-level test, you'll need to compare the test statistic with the critical value.

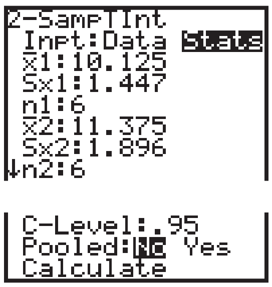
Calculator Note 9F: Two-Sample *t*- and *z*-Intervals

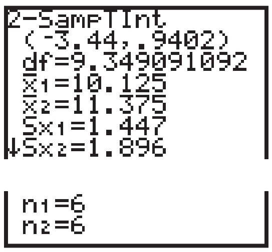
2-SampTInt and 2-SampZInt

Confidence Interval for the Difference between Two Means When σ_1 and σ_2 Are Unknown

The TI-83 Plus and TI-84 Plus calculate a confidence interval for the difference between two means when σ_1 and σ_2 are unknown with the command **2-SampTInt**. You find the command by pressing STAT, arrowing over to **TESTS**, and selecting **0:2-SampTInt**. You can use data saved in lists or enter the sample means, sample standard deviations, and sample sizes yourself. Either way, you must enter the confidence level desired. **Calculate** outputs the confidence interval and degrees of freedom. Note that **2-SampTInt** gives you the choice of **pooled**. As stated in the student book, pooling should be used only when you have been told that $\sigma_1 = \sigma_2$. These screens show the calculations for the walking-babies example from pages 621–622 of the student book.







Confidence Interval for the Difference between Two Means When σ_1 and σ_2 Are Known

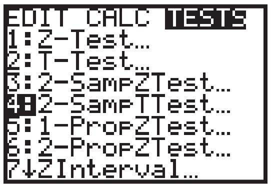
The TI-83 Plus and TI-84 Plus calculate a confidence interval for the difference between two means when σ_1 and σ_2 are known with the command **2-SampZInt**. You find the command by pressing [STAT], arrowing over to **TESTS**, and selecting **9:2-SampZInt**. As for **2-SampTInt**, you can use data saved in lists and enter σ_1 and σ_2, or you can enter the population means, sample means, and sample sizes yourself. Either way, you must enter the confidence level desired. **Calculate** outputs the confidence interval.

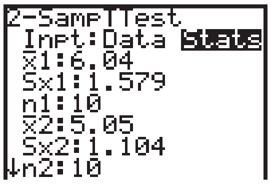
Calculator Note 9G: Two-Sample *t*- and *z*-Tests

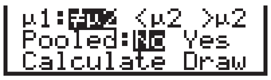
2-SampTTest and 2-SampZTest

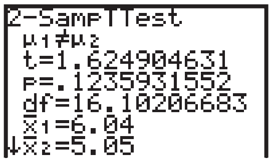
Significance Test for the Difference between Two Means When σ_1 and σ_2 Are Unknown

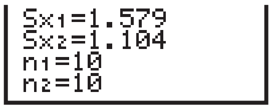
The TI-83 Plus and TI-84 Plus calculate a confidence interval for the difference between two means when σ_1 and σ_2 are unknown with the command **2-SampTTest**. You find the command by pressing [STAT], arrowing over to **TESTS**, and selecting **4:2-SampTTest**. You can use data saved in lists or enter the sample means, sample standard deviations, and sample sizes yourself. Either way, you must select whether the test is one-sided or two-sided. **2-SampTInt** gives you the choice of **pooled**. As stated in the student book, pooling should be used only when you have been told that $\sigma_1 = \sigma_2$. **Calculate** outputs the values of t and p and the degrees of freedom. **Draw** gives a shaded distribution. These screens show the calculations for the aldrin data example from pages 626–627 of the student book.

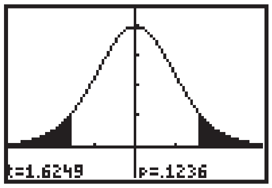












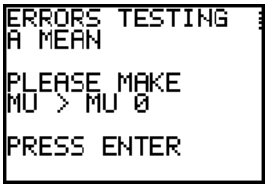
Confidence Interval for the Difference between Two Means When σ_1 and σ_2 Are Known

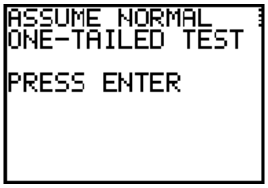
The TI-83 Plus and TI-84 Plus calculate a confidence interval for the difference between two means when σ_1 and σ_2 are known with the command **2-SampZTest**. You find the command by pressing [STAT], arrowing over to **TESTS**, and selecting **3:2-SampZTest**. As for **2-SampTTest**, you can use data saved in lists or enter the population means, sample means, and sample sizes yourself. Either way, you must select whether the test is one-sided or two-sided. **Calculate** outputs the values of t and p and the degrees of freedom. **Draw** gives a shaded distribution.

Calculator Note 9H: Types of Errors, Revisited

If you would like to further explore Type II errors and the power of a significance test for a mean, you can use the program ERRORS2. The program graphically shows the effects that changing parameters has on the probability of a Type II error and the power of the test. This program behaves similarly to the program ERRORS that was introduced in Chapter 8.

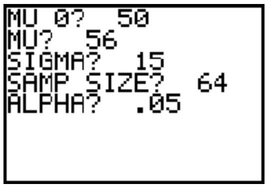
a. Run the program: press PRGM and select **ERRORS2** from the **EXEC** menu. Press ENTER. Read the introductory screens, which emphasize that the program uses a normal approximation and a one-tailed test where $\mu > \mu_0$. Press ENTER after each screen.



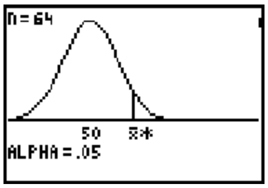


b. At the prompts, enter the hypothesized mean for the population mean, μ_0 (**MU 0**), the true value of the population mean, μ (**MU**), the population standard deviation, σ (**SIGMA**), the sample size, and the significance level, α (**ALPHA**). The true mean can be any value greater than the hypothesized mean. Press ENTER after each value.

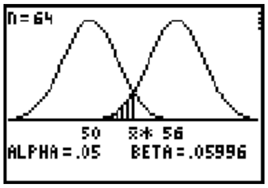
As an example, here you test H_0: $\mu = 50$ when, in fact, $\mu = 56$. Your sample size will be 64, and you are using $\alpha = 0.05$.



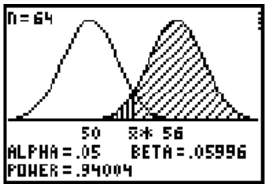
c. The program first graphs a sampling distribution for the hypothesized population mean. The critical value is labeled as $\bar{x}^*$. If $\bar{x} > \bar{x}^*$, you will (correctly) reject H_0. If $\bar{x} < \bar{x}^*$, you will (incorrectly) fail to reject H_0.



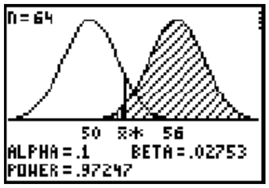
d. Press [ENTER] to graph a sampling distribution for the true population mean. The tail is highlighted below the critical value $\bar{x}^*$, which represents the probability of not rejecting this false null hypothesis. The area of this tail, called BETA, or β, represents the probability of a Type II error.



e. Press [ENTER] again to shade the second curve above the critical value. This area, $1 - \beta$, represents the power of the test. It represents the probability of rejecting this false null hypothesis.



f. Press [ENTER] to get options to change the true population mean, the sample size, or the significance level. Changing the parameters shows the effects on the probability of a Type II error. For example, if you increase the significance level, you see that the probability of a Type II error decreases. (However, if you were correct in the first place that $\mu = 50$, you have increased the probability of a Type I error.)



In practice you don't know the value of μ, so it is impossible to compute the probability of a Type II error and the power of the test. However, given a plausible value of μ, ERRORS2 allows you to explore how the probability of a Type II error and the power of the test are affected by changes in σ, α, and the sample size.

The text of the program is given on the next page.


```
PROGRAM:ERRORS2
ClrHome
PlotsOff
FnOff
AxesOn
Disp "ERRORS TESTING"
Disp "A MEAN"
Disp ""
Disp "PLEASE MAKE"
Disp "MU > MU 0"
Disp ""
Disp "PRESS ENTER"
Pause
ClrHome
Disp "ASSUME NORMAL"
Disp "ONE-TAILED TEST"
Disp ""
Disp "PRESS ENTER"
Pause
ClrHome
Input "MU 0?   ",M
Input "MU?   ",A
If M ≥ A
Then
Disp "PLEASE MAKE"
Disp "MU > MU 0"
Disp "MU 0   ",M
Input "MU?   ",A
End
Input "SIGMA?   ",S
Input "SAMP SIZE?   ",N
Input "ALPHA?   ",L
Lbl D
"0"→Y1
"normalpdf(X,M,S/√(N))"→Y2
"normalpdf(X,A,S/√(N))"→Y3
FnOff 3
(M-3*S/√(N))→Xmin
(A+3*S/√(N))→Xmax
0→Xscl
(Y2(M)+.02)→Ymax
-Ymax/2→Ymin
0→Yscl
ClrDraw
Line(Xmin,0,Xmax,0)
DispGraph
Text(0,0,"N = ",N)
Text(43,round((91*M-91*Xmin)/(Xmax-
Xmin),0)-2,M)
(invNorm(1-L,M,S/√(N))→J
Line(J,0,J,Y2(J))
Text(43,round((91*(J-Xmin)/(Xmax-
Xmin)),0)," x̄* ")
DrawF Y2
Text(50,0,"ALPHA = ",L)
Pause
FnOn 3
Text(43,round((91*A-91*Xmin)/(Xmax-
Xmin),0)-2,A)
Line(J,0,J,Y3(J))
Shade(0,Y3,J-3*S/√(N),J,1,2)
normalcdf(-1E699,J,A,S/√(N))→B
round(B,5)→B
Text(50,47,"BETA = ",B)
Pause
Shade(0,Y3,J,1E610,4,4)
Text(57,0,"POWER = ",1-B
Pause
ClrHome
Menu("CHANGE:","MU",A,"SAMP SIZE",B,
"ALPHA",C,"QUIT",E)
Lbl A
Input "NEW MU?   ",A
If M ≥ A
Then
Disp "PLEASE MAKE"
Disp "MU > MU 0"
Disp "MU 0   ",M
Input "NEW MU?   ",A
End
Goto D
Lbl B
Input "NEW SAMP SIZE?   ",N
Goto D
Lbl C
Input "NEW ALPHA?   ",L
Goto D
Lbl E
Clr
```