

6

Probability Distributions

Calculator Note 6A: Computing Expected Value, Variance, and Standard Deviation from a Probability Distribution Table

Using Lists to Compute Expected Value, Variance, and Standard Deviation

You can practice computing the expected value, variance, and standard deviation of a probability distribution table by using the spreadsheet capabilities of the List Editor. For example, consider the data in Display 6.11 and the calculations on pages 366–367 of the student book.

- Enter the probability distribution table into lists L1 and L2. (A representative number is chosen for each interval of x .)

L1	L2	L3	Z
0	.06		
3	.28		
8	.25		
15.5	.21		
35.5	.17		
55	.03		

L2Σ1 = .06			

- Define list L3 as the products of the values and the probabilities.

L1	L2	L3	# 3
0	.06	0	
3	.28	.84	
8	.25	2	
15.5	.21	3.255	
35.5	.17	6.035	
55	.03	1.65	

L3 = "L1*L2"			

- The expected value is the sum of list L3. On the Home screen, calculate $\text{sum}(L3)$. Find the $\text{sum}()$ command by pressing 2ND [LIST], arrowing over to MATH, and selecting 5:sum(.

sum(L3)	13.78
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- Define list L4 as the products of the squares of the deviations from the mean and the probabilities, $(L1 - 13.78)^2 * L2$.

L2	L3	#	L4	# 4
.06	0		11.393	
.28	.84		32.538	
.25	2		8.3521	
.21	3.255		62.126	
.17	6.035		80.199	
.03	1.65		50.873	

L4 = "(L1-13.78)^2*L2"				

- e. The variance is the sum of list L4. The standard deviation is the square root of the variance.

```
SUM(L4) 184.0766
√(Ans) 13.56748319
```

Using a Calculator Command to Find Expected Value, Variance, and Standard Deviation

More directly, the expected value and standard deviation can be calculated using the 1-Var Stats command. Find this command by pressing [STAT], arrowing over to CALC, and selecting 1:1-Var Stats. For a frequency table, you follow the command with the names of two lists, separated by a comma. The first list you enter must contain the values, and the second list must contain the relative frequencies.

```
1-Var Stats L1,L2
```

```
1-Var Stats
x̄=13.78
Σx=13.78
Σx²=373.965
Sx=
σx=13.56748319
n=1
```

Note that the calculator displays no value for S_x when list L2 contains relative frequencies. The calculator assumes that you are summarizing a probability distribution for a population and therefore omits the sample standard deviation.

Calculator Note 6B: Binomial Probabilities **binompdf(**

The TI-83 Plus and TI-84 Plus can calculate binomial probabilities in two ways.

- Use the formula $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$ and enter each factor individually. To get the binomial coefficient $\binom{n}{k}$, first enter the number to choose from, then press [MATH], arrow over to PRB, select 3:nCr, and then enter the number to choose.

```
MATH NUM CPX PRB
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

```
(7 nCr 3)(.27)^3
(.73)^4
.1956369022
```

- Use the binomial probability density function. You find the **binompdf(** command by pressing [2ND] [DISTR] and selecting **binompdf(** from the DISTR menu. The syntax is **binompdf(n, p, k)**. This command calculates the binomial probability for k successes out of n trials when the probability of success on any one trial is p .

```

DISTR DRAW
5:tcdf(
6:X²Pdf(
7:X²cdf(
8:FPdf(
9:Fcdf(
0:binompdf(
1:binomcdf(
    
```

```

binompdf(7,.27,3)
.1956369022
    
```

The number of successes can also be entered as a list of numbers set in braces.

```

binompdf(7,.27,{
3,4})
(.1956369022 .0...
    
```

If k is omitted, the entire binomial probability distribution is displayed. You can scroll through the list of probabilities using the left and right arrow keys. You could also store the probabilities in a list. There are three ways to do this: press **STO** and the name of a list after the `binompdf` command; press **STO** and the name of a list immediately after calculating the `binompdf` (the Home screen will automatically begin the entry line with `Ans`); or define the list with the `binompdf` command in the List Editor.

```

binompdf(7,.27)→
-1
(.1104739852 .2...
    
```

L1	L2	L3	1
.1104739852	-----	-----	
.28602			
.31737			
.19564			
.07236			
.01606			
.00198			
L1(1)=.1104739851...			

Note that a binomial distribution is a discrete distribution, as opposed to a normal distribution, which is continuous. Therefore, `binompdf` cannot be used in the `Y=` screen, though `normalpdf` can. For example, $Y_1 = \text{binompdf}(7,.25,X)$ does not result in a graph. See Calculator Note 6D for instructions about graphing a binomial distribution.

Calculator Note 6C: Binomial Cumulative Distribution `binomcdf`

```

binomcdf(7,.27,3)
.9094991052
    
```

To calculate the cumulative probability of a binomial distribution, use the binomial cumulative distribution function. Find this command by pressing **2ND** **[DISTR]** and selecting `binomcdf` from the `DISTR` menu. The syntax is `binomcdf(n, p, k)`. For example, for the example on pages 384–385 of the student book, `binomcdf(7,.27,3)` gives the probability of fewer than three adults with a bachelor’s degree among seven randomly chosen adults (or the cumulative probability of zero, one, or two bachelor’s degrees).

To find the probability of three or more bachelor’s degrees, subtract the result from 1.

Calculator Note 6D: The Shape, Center, and Spread of a Binomial Distribution

If you enter all values of X into list L1 and the binomial probabilities of k successes out of n trials into list L2, you can use the command 1-Var Stats L1,L2 to calculate the expected value and standard deviation of the binomial distribution. This example shows a binomial distribution with $n = 20$ and $p = 0.7$. Calculator Note 5B showed you how to use the sequence command to enter a sequence of integers.

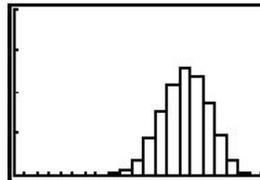
```
Seq(X,X,0,20,1)→
L1
{0 1 2 3 4 5 6 ...
binomPdf(20,.7)→
L2
{3.486784401e-1...
```

L1	L2	L3	Z
0	3.486784401e-1	-----	
1	1.6E-3		
2	3.6E-8		
3	7E-7		
4	6E-6		
5	3.7E-5		
6	2.2E-4		
L2(1)=3.486784400...			

```
1-Var Stats
x̄=14
Σx=14
Σx²=200.2
Sx=
σx=2.049390153
n=1
```

Once the binomial distribution is entered into two lists, you can display the distribution in a histogram. This histogram is the same as the lower-right plot in Display 6.28 on page 389 of the student book. (Note: In order to duplicate the histogram as it appears in the student book, be sure to set the window to $[0, 21, 1, 0, 0.3, 0.075]$, especially setting $Xscl = 1$.)

```
Plot1 Plot2 Plot3
On Off
Type: L1 L2 L3
Xlist: L1
Freq: L2
```



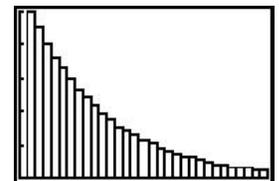
$[0, 21, 1, 0, 0.3, 0.075]$

Calculator Note 6E: Graphing Geometric Distributions

The probabilities for a finite sequence of values of k can be calculated, stored in the List Editor, and graphed, much as described in Calculator Note 6D. Here is the geometric distribution for $p = 0.1$, as shown in the first plot in Display 6.29 on page 394 of the student book. The command `geometpdf(` is explained in Calculator Note 6F.

```
Seq(X,X,1,30,1)→
L1
{1 2 3 4 5 6 7 ...
geometpdf(.1,L1)→
L2
{.1 .09 .081 .0...
```

L1	L2	L3	Z
1	.1	-----	
2	.09		
3	.081		
4	.0729		
5	.06561		
6	.05905		
7	.05314		
L2(1)=.1			



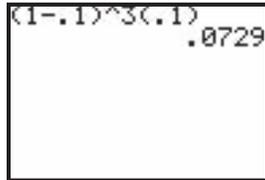
$[0, 31, 1, 0, 0.1, 0.02]$

Calculator Note 6F: The Geometric Distribution

geometpdf(

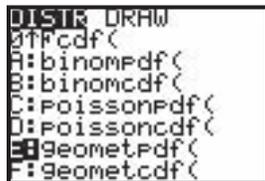
Similar to binomial probabilities, geometric probabilities can be calculated in two ways.

1. Use the formula $P(X = k) = (1 - p)^{k-1}p$.



```
(1-.1)^3(.1)  
.0729
```

2. Use the geometric probability density function. Find the `geometpdf(` command by pressing `2ND` [DISTR] and selecting `geometpdf(` from the DISTR menu. The syntax is `geometpdf(p, k)`. This command calculates the probability that the first success occurs on the k th trial, where p is the probability of each success.

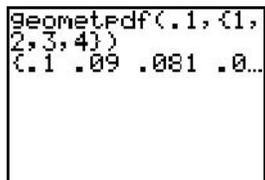


```
DISTR DRAW  
A:binompdf(  
B:binomcdf(  
C:Poissonpdf(  
D:Poissoncdf(  
E:geomet.pdf(  
F:geomet.cdf(  
|
```



```
geomet.pdf(.1,4)  
.0729
```

The parameter k can also be entered as a list of numbers set in braces.

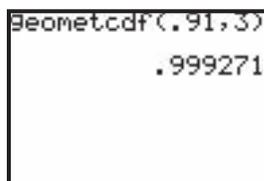


```
geomet.pdf(.1,{1,  
2,3,4})  
.1 .09 .081 .0729
```

Because any geometric distribution has an infinite number of values, k cannot be omitted.

Calculator Note 6G: Geometric Cumulative Distribution `geometcdf(`

To calculate the cumulative probability of a geometric distribution, use the geometric cumulative distribution function. Find this command by pressing `2ND` [DISTR] and selecting `geometcdf(` from the DISTR menu. For example, for P29b on page 400 of the student book, `geometcdf(.91,3)` gives the probability of the first nondefective engine before the fourth trial (or the probability of success on the first, second, or third trial).



```
geomet.cdf(.91,3)  
.999271
```